GW170817: Light curve and spectra
The Lazy Theorist's View

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I. RELEVANT OBSERVATIONS

- The spectrum has a peak at 1.1 microns with a half-max half width of ≈ 0.1 microns
- The light curves have a rise time and decay timescale of 1–2 days
- The peak bolometric luminosity is a few 10^{41} erg/s

II. CONCLUSIONS

Assuming that the spectral peak at 1.1 micron is intrinsically a delta function, and the $\sim 10\%$ broadening is due to the ejecta velocity v, the upper limit on v is approximately $v \sim 0.1c$.

Using this value of velocity and the light curve rise and decay time of 1–2 days or $t_{\rm LC} \sim 10^5$ seconds (see Eq. 5) allows us to solve for κM. The ejecta mass is then

$$M \sim 5 \times 10^{-3}\, M_\odot \left(\frac{0.1\,\mathrm{g\ cm}^{-2}}{\kappa}\right).$$

As much as $\frac{GM_{\rm NS}^2}{30\,\rm km} \sim 10^{53}$ erg of binding energy were deposited in the merger. However, for the values of ejecta mass and velocity given above, only $E_0 = 0.5Mv^2 \sim 5 \times 10^{49}$ erg of energy are in the ejecta (the rest are, presumably, in neutrinos, in the rotational energy of the post-merger remnant, etc.).

Of course, the ejecta mass could be a factor of 10 larger/smaller if κ is a factor of 10 smaller/larger than assumed. However, all models appear to require continued energy input through radioactive decay. The peak luminosity for diffusion through an expanding medium (Eq. 6) is far smaller than observed for any reasonable value of κ:

$$L \sim \frac{R_0 E_0 c}{\kappa M} \sim \frac{30\,\mathrm{km}\, v^2 c}{\kappa} \sim 10^{37} \mathrm{erg\ s}^{-1} \left(\frac{0.1\,\mathrm{g\ cm}^{-2}}{\kappa}\right).$$

The total heating output of radioactive decay ϵ can be parametrized as $\epsilon \equiv fMc^2$, with f given by (Eq. 7):

$$f \sim 10^{-6} \frac{L_{\rm peak}}{10^{41}\mathrm{erg\ s}^{-1}} \frac{0.005 M_\odot}{M}.$$

III. GENERAL BACK-OF-THE-ENVELOPE THEORY

We will follow Arnett's classical papers from the early 1980s in assuming a one-zone spherical model with constant temperature T, density ρ, and opacity κ in an expanding cloud of mass M of radius R, following an initial injection of energy E_0 when the gas cloud has radius R_0. The cloud expands homologously, with velocity $v = \sqrt{2E_0/M}$ (i.e., most of the energy goes into the kinetic motion of the cloud), so that the cloud's volume is $V = 4/3\pi R^3$, where $R = vt$ (in particular, $R_0 = vt_0$). Within the cloud, the radiation energy density dominates the gas energy, so the internal energy density is $u = aT^4$ and radiation pressure is $p = a/3T^4 = u/3$.

Energy conservation yields

$$\dot{E} = -p\dot{V} + \dot{\epsilon} - L, \qquad (1)$$

where $\dot{\epsilon}$ is the rate of radioactive heating and L is the luminosity. We simplify the diffusion equation

$$L(r) = -4\pi r^2 \frac{c}{3\kappa\rho} \frac{\partial u(r)}{\partial r}$$

as

$$L \approx 4\pi R^2 \frac{c}{3\kappa\rho}\frac{u}{R} = \frac{cV^2 u}{\kappa M R^2}. \tag{2}$$

If $\dot{\epsilon} = 0$, then $\dot{E} + p\dot{V} + L = 0$ and we can substitute L from above along with $\dot{E} = u\dot{V} + V\dot{u}$, with $\dot{V} = 3Vv/R$, to obtain

$$\frac{\dot{u}}{u} = -4\frac{1}{t} - \frac{4\pi vc}{3\kappa M}t. \tag{3}$$

This can be integrated to yield

$$u(t) = u_0(t/t_0)^{-4}\exp\left[-\frac{2\pi cv}{3\kappa M}t^2\right], \tag{4}$$

where $u_0 = u(t_0) = E_0/V_0$.

Therefore, the light curve decay timescale is

$$t_{\rm LC} \sim \sqrt{\frac{\kappa M}{cv}}. \tag{5}$$

Note that this is the geometric mean of the diffusion timescale $t_{\rm diff} = R_0/(c\tau) \sim \frac{M\kappa}{cR_0}$ [where $\tau \sim \kappa\rho R_0$ is the optical depth of the initial cloud] and the expansion timescale $t_{\rm exp} = R_0/v$, as the photons diffuse through an expanding medium with changing optical depth. The decay timescale $t_{\rm LC}$ is also roughly the time after the merger when the light curve reaches the peak, which can be estimated by determining the time required for the cloud to reach a radius R from which photons can diffuse on a comparable timescale, $t_{\rm diff} = t_{\rm exp}$.

The luminosity at time $t_{\rm LC}$ is approximately

$$L_{\rm LC} \sim \frac{cR_{\rm LC}^4}{\kappa M}u_{\rm LC} \sim \frac{cR_0^4}{\kappa M}\frac{E_0}{V_0} \sim \frac{cR_0 E_0}{\kappa M}. \tag{6}$$

This is a rather small number: if the initially injected energy is of order the gravitational binding energy of the gas cloud, $E_0 \sim \frac{GM_{\rm BH}M}{R_0}$, then the peak diffusion-limited luminosity of an expanding gas cloud is of order the Eddington luminosity:

$$L_{\rm LC} \sim \frac{cGM_{\rm BH}}{\kappa}.$$

The way to get a larger luminosity is to increase the energy production. At peak luminosity, $dL/dt = 0$; this implies $L_{\rm peak} = \dot{\epsilon}(t_{\rm peak})$. Assuming that the luminosity reaches its peak value long after the peak of the r-process radioactive decay rate (generally true because the luminosity is still diffusion-limited when radioactive decay peaks), the peak luminosity will be observed at time $t \sim t_{\rm LC}$:

$$L_{\rm peak} \sim \frac{\epsilon}{t_{\rm LC}}, \tag{7}$$

where ϵ is the total radioactive heating, which can be parametrized as $\epsilon \equiv fMc^2$.

Later addition (see, e.g., Waxman et al., 2017): Following the peak, the luminosity will initially decay, roughly following $\dot{\epsilon}(t)$, as the ejecta become optically thin and the diffusion time scale grows shorter. Although the energy injection rate from an individual radioactive source decays exponentially, the mixing of multiple species can lead to a power-law decay of injected energy. For example, if the radioactive decay time scales are distributed following $p(t_{\rm decay}) \propto t_{\rm decay}^{-2}$, then

$$\dot{\epsilon}(t) \propto \int_{t_{\rm decay,min}}^{t_{\rm decay,max}} p(t_{\rm decay})\exp(-t/t_{\rm decay})dt \propto 1/t \tag{8}$$

for $t_{\rm decay,min} \ll t \ll t_{\rm decay,max}$.

If the bulk of the energy is initially deposited into the kinetic energy of electrons, the $L \sim \dot{\epsilon}$ behaviour will change once electrons can escape from the gas cloud without depositing their energy to be re-processed into photons. The "optical thickness" for electrons is $\tau_{\rm e} \approx \kappa_{\rm e}\rho R \approx M\kappa_{\rm e}/v^2/t^2$. When $\tau_{\rm e} \ll 1$, only a fraction $\tau_{\rm e}$ of the entire radioactive energy input would be converted into photons, so the luminosity would follow $\tau_{\rm e}\dot{\epsilon}(t)$; e.g., for $\dot{\epsilon}(t) \propto 1/t$, the luminosity would scale as $1/t^3$.